5 Comparing Apples to Oranges

Comparing Functions Using Different Representations

WARM UP

1. Determine the slope described by the table of values shown.

x	y
−1	15
0	25
2	45
5	75

2. Determine the slope described by the equation $-12x + 2y + 30 = 0$.
3. Determine the hourly rate of change described in the situation given. Jane is a tutor and is paid $20 for a half hour session.

LEARNING GOALS

- Compare properties of two functions, each represented in a different way (equation, table, context, or graph).
- Compare the slopes of two functions, each represented in a different way.

You have represented functions as ordered pairs, mappings, sequences, tables, equations, and graphs. How can you compare functions when they are displayed using different representations?

Getting Started

Comparing Apples to Apples

Examine each set of functions and determine which has the greater rate of change, if either. Explain your reasoning.

1. Table A

x	y
−2	−8
2	−5
6	−2

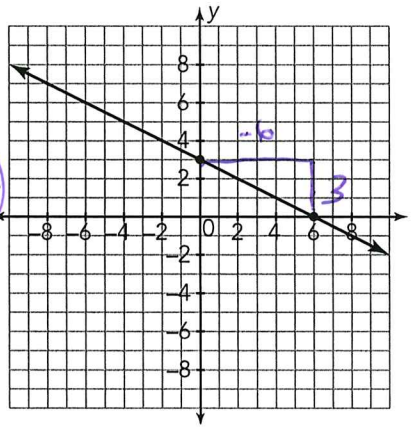
$m = \frac{-5 - -8}{2 - -2} = \frac{3}{4}$

Table B Greater

x	y
−5	−46
1	−38
7	−30

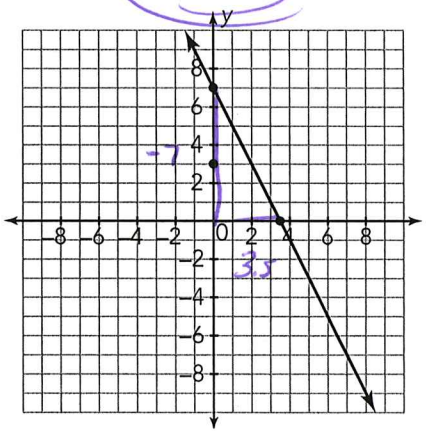
$m = \frac{-38 - -46}{1 - -5}$ $\frac{8}{6} = \frac{4}{3}$

2. Graph A



$\frac{3}{-6} = -\frac{1}{2}$

Graph B Greater



$\frac{-7}{3.5} = -2$

3. Equation A Greater

$5x + 6y = 60$

$m = -\frac{5}{6}$

$\frac{6y}{6} = \frac{60}{6} - \frac{5x}{6}$

$y = 10 - \frac{5}{6}x$

Equation B

$y = -\frac{1}{4}x - 2$

4. An ice cream shop is choosing a milk delivery service. The Spotted Cow charges $2.80 per gallon, plus a $2 delivery fee. Dairy Farms charges $2.10 per gallon, plus a $10 delivery fee.

Spotted Cow is greater.

ACTIVITY
5.1 Comparing Linear Functions

You have worked with many linear functions presented in real-world and mathematical problems. You have also represented various linear functions through equations, tables, and graphs. In this lesson, you will compare the rates of change in different representations of two or more linear functions.

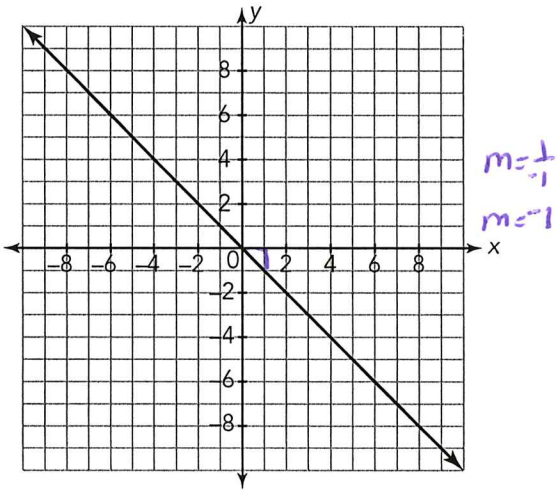
In Questions 1 through 3, analyze the two distinct linear functions. Identify which function has the greater rate of change. Explain your reasoning.

1. Function A

$y = 8x - 3$ greater

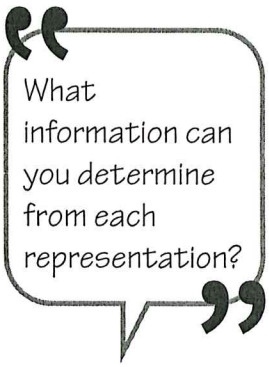
$m = 8$

Function B



$m = \frac{1}{-1}$

$m = -1$

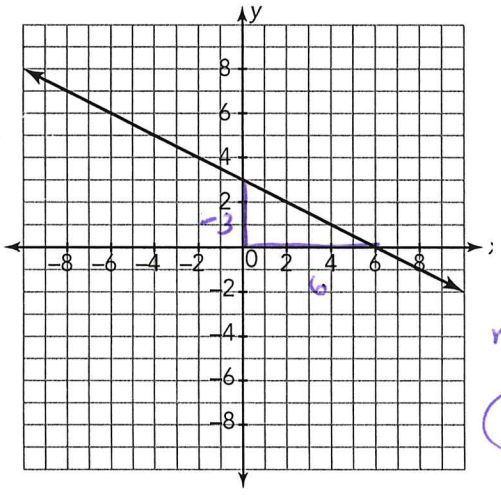


2. Function C greater

x	y
−1	−6
0	−3
2	3
5	12

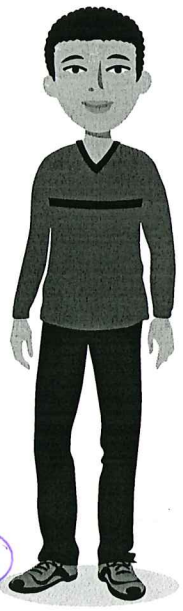
$m = \frac{12-3}{5-2} = \frac{9}{3} = 3$

Function D



$m = \frac{-3}{6}$

$m = -\frac{1}{2}$



NOTES

3. **Function E**

$6x + y = 1$

$y = 1 - 6x$

$m = -6$

Function F

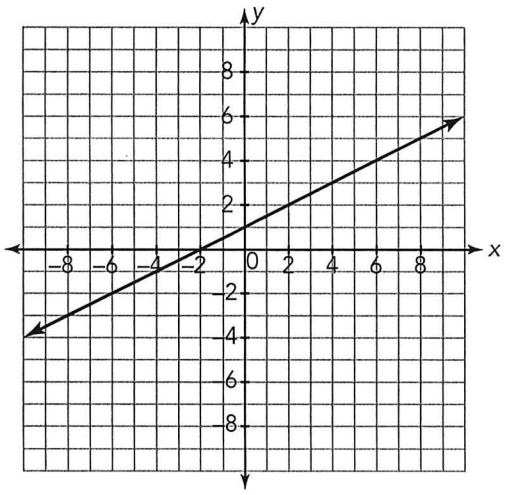
x	y
−1	−6
1	6
3	18
5	30

They are both 6.

$\frac{18-6}{3-1} = \frac{12}{2} = 6$

4. **Alicia, Cherie, and John had been studying rates of change and were discussing the best way to determine which linear function has the greater rate of change.**

Function G



Function H

x	y
0	1
1	3
2	5
3	7

Alicia

I can identify two points from the graph and two points from the table and use the formula $\frac{y_2 - y_1}{x_2 - x_1}$ to calculate the slope.

Function G: (0, 1) and (2, 2)

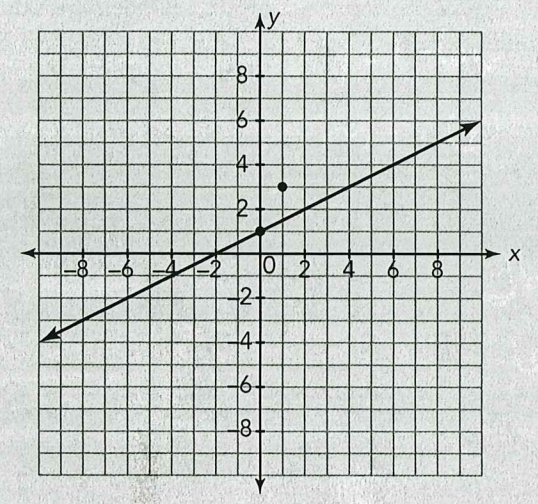
$m = \frac{2-1}{2-0} = \frac{1}{2}$

Function H: (0, 1) and (1, 3)

$m = \frac{3-1}{1-0} = 2$

Function H has a rate of change of 2, which is greater than Function G's rate of change of $\frac{1}{2}$.

Cherie



I started plotting Function H on the graph with Function G. I noticed that the functions have the same y-intercept but Function H is steeper than Function G. This means that Function H has a greater rate of change than Function G.

John

I can see from the graph of Function G that the vertical distance increases by one unit for every two units of increase in the horizontal distance. The rise/run is $\frac{1}{2}$, so the rate of change of Function G is $\frac{1}{2}$.

In the table, as the x-values increase by 1, the y-values increase by 2. The rate of change of Function H is $\frac{2}{1} = 2$. Function H has the greater rate of change.

a. How is Cherie's method different from John's and Alicia's methods?

Cherie didn't calculate the slope, she observed the graph.

b. Compare John's method to Alicia's method.

Alicia used 2 points & calculated slope.
John observed the data & calculated the slope.

c. Which method is the most efficient in this situation?

using the formula is most accurate.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

5. John and Alicia then encountered the two linear functions shown. Again, they wanted to determine which function had the greater rate of change.

   Alicia said that it was necessary to use a formula to calculate the rate of change for Function I and to rearrange Function J into the slope-intercept form of a linear equation.

   John said that would take too much time. He says he only needs to rewrite Function J in slope-intercept form.
   Who is correct?

Function I

x	y
−4	4
−2	4
0	4
2	4

Function J

$4x - y = 0$

ACTIVITY
5.2 Comparing Functions in Context

Read each problem situation and use the different representations to answer the questions.

1. **Charlie is an avid reader and purchases e-books. For his birthday, his grandparents want to enroll him in a book of the month club. They plan to purchase a $100 gift certificate to the e-book club. In their research, they found two plans with comparable book offerings.**

   Readers-R-Us automatically loads the book of the month onto the e-reader at the beginning of each month, and for this service, the club charges each member $4.50 per month. Consider an equation in which *y* represents the amount of money remaining in Charles's account and is expressed as a function of the number of months Charlie is a member.

   $y = 100 - 4.50x$

   A second company, Bookworms, presents the table shown to illustrate their plan, given a purchase of a $100 gift certificate.

Months	Balance ($)
0	100
5	75
10	50
15	25

   a. **Identify which function has the greater rate of change. Explain your reasoning.**

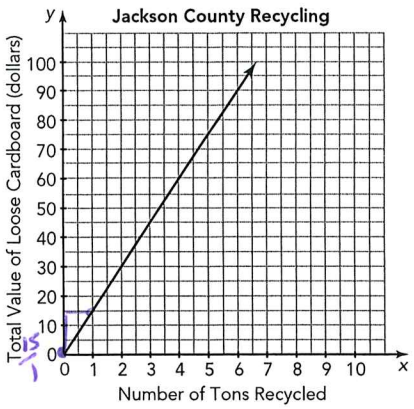
   Bookworm = $5 per book

   $\frac{75-100}{5-0} = \frac{-25}{5} = -5$

   $y = -5x + 100$

   b. **Which plan should Charlie's grandparents choose? Explain your reasoning.**

   Readers R us because the cost per book is lower.



2. Washington County Recycle Center currently pays $20 per ton of loose cardboard. Jackson County Recycling represents their pay rate for cardboard using the graph shown. $15 per ton

After moving from one county to the other, Lashonda needs to recycle her moving boxes. If Lashonda wants to earn the most money possible for her cardboard, which recycling center should she choose? Explain your reasoning.

She should go to Washington County - she gets more money per ton.

3. Bobby's Recycle Center currently pays $1.59 per pound of aluminum cans. Bobby needs to write a formula to enter into his spreadsheet to keep a record of how much he has paid for cans. Consider a formula in which *y* represents the total value of the aluminum cans and is expressed as a function of the number of pounds of recycled aluminum cans. He entered the following into his spreadsheet:

	A	B	C
1	POUNDS of CANS	AMOUNT PAID	
2	1	=A2+1.59	
3	2		
4	3		
5	4		
6			

a. Is Bobby's formula correct? Explain your reasoning.

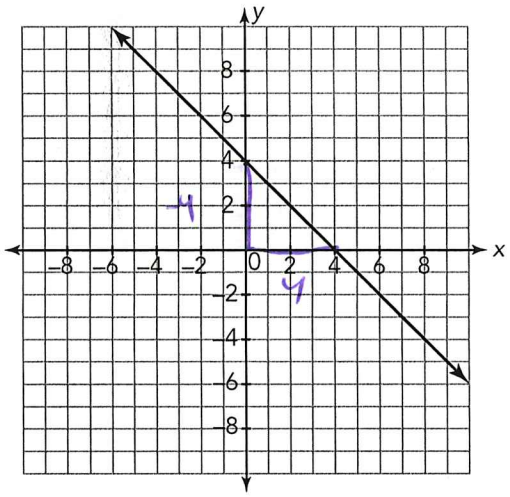
No - it should be multiplication not addition.

b. Will the spreadsheet show that he has paid out more or less than his actual pay-outs? Explain using your knowledge of rates of change.

ACTIVITY
5.3 Ordering Rates of Change

1. **Consider each representation of four distinct linear functions. Order the functions from least to greatest rate of change. Justify your ordering.**

Function A



$m = -\frac{4}{4} = -1$

Function B

$4y + x = 12$

$\frac{4}{4}y = \frac{12}{4} - \frac{x}{4}$

$y = 3 - \frac{1}{4}x$

$m = -\frac{1}{4}$

$\frac{x}{4} = \frac{1}{4}x$

$\frac{12}{4} = 3$

Function C

x	y
−2	4
0	7
2	10
4	13

$\frac{13-10}{4-2} = \frac{3}{2}$

Function D

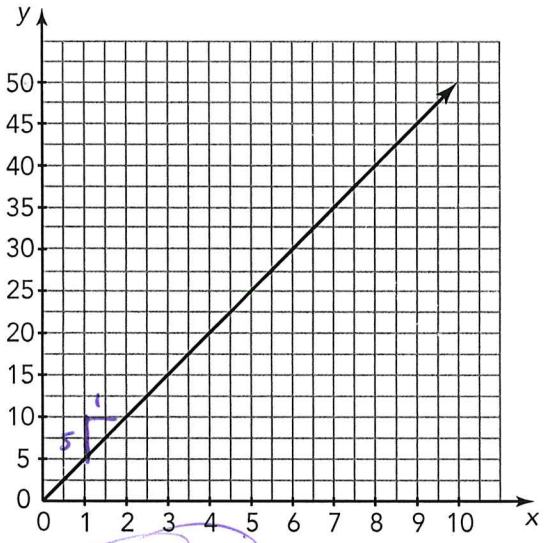
The Used Book Store will pay $0.50 for each box of hardcover books.

$m = 0.50$

B, D, A, C - least to greatest absolute value.

2. **Each linear function shown describes the steepness of the initial climb of the roller coaster track. For each representation, let *y* represent the height of the coaster in feet, and let *x* represent the horizontal distance in feet. List the roller coasters in order with respect to the steepness.**

Jack Rabbit



$m = \frac{5}{1}$

Racer

$2y - 9x = 4$

$\frac{2y}{2} = \frac{4}{2} + \frac{9x}{2}$

$y = 2 + \frac{9}{2}x$

$m = \frac{9}{2} = 4.5$

Pippin

x	y
0	2
1	7.5
3	18.5
5	29.5

$\frac{7.5 - 2}{1 - 0} = \frac{5.5}{1} = 5.5$

Thunderbolt

The track rises 3 feet per 1 horizontal foot.

$\frac{3}{1} = 3$

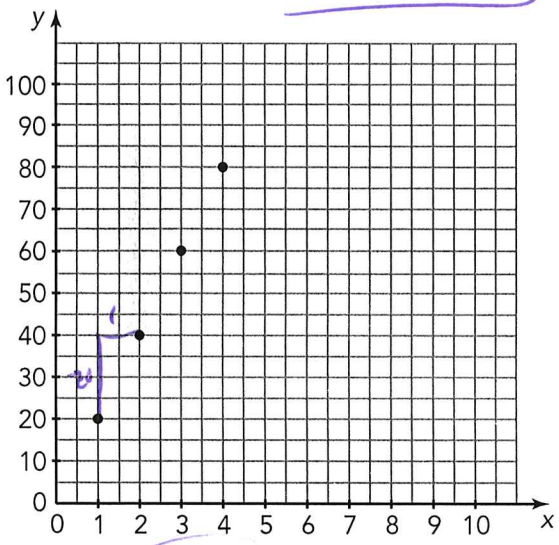
Thunderbolt
Racer
Jack Rabbit
Pippin

3. The linear functions shown describe the bank accounts of four students. The *y*-values represent the dollar amounts in the bank accounts, and the *x*-values represent the time in months since September 1st. If each student continues to save money at the constant rate shown, who will be the first to save $500? Justify your response.

$y = 20x$ $\quad \frac{500}{20} = \frac{20x}{20}$

D'Andre

25 months



$m = \frac{20}{1} = 20$

25 months to save $500

Fiona

$y - 12x = 100$

$y = 100 + 12x$

$m = 12$

34 months to get $500

$500 = 100 + 12x$

$-100 \quad -100$

$\frac{400}{12} = \frac{12x}{12}$

33.3 months

Sam

x	y
–1	45
0	60
1	75
2	90

30 months to get $500

$y = 15x + 60$

$500 = 15x + 60$

$-60 \quad -60$

$\frac{440}{15} = \frac{15x}{15}$

29.3 months

$\frac{60 - 45}{0 - -1} = \frac{15}{1} = 15$

Michelle

Michelle opened her bank account on September 1st with $25 and continues to deposit $25 each month.

19 months to get $500

$m = 25$

$y = 25x + 25$

$500 = 25x + 25$

$-25 \quad -25$

$\frac{475}{25} = \frac{25x}{25}$

19 months

Michelle

John

Fiona – she starts with more than Sam

Sam

NOTES

TALK the TALK

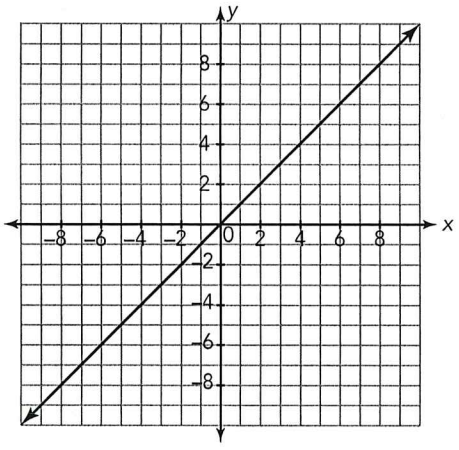
The Whole Fruit Basket

1. Create a situation to represent the table of values shown.

x	−1	0	2	5
y	5	10	20	35

2. Write an equation that will have a slope that is less steep than the relationship in the table.

3. How does the slope in this graph compare to the slope in Questions 1 and 2?



4. What strategies did you use to create your linear functions and to compare the slopes?

Assignment

Write

Explain how to determine the slope from a table, from an equation in any form, from a context, and from a graph.

Remember

Information visible in a table, context, equation, or graph often can be used to compare slopes without determining the actual values for the slopes.

Practice

1. Shawna is a professional dog walker. She offers two different payment plans. In each plan, she agrees to walk your dog twice a day for at least one mile per walk. Suppose you want to employ Shawna but need to choose between the two payment plans.
   The first plan charges a rate of $5 per day.
   The second plan is described using a table of values, and you must purchase 20 days' worth of services per month.

Days	Cost ($)
20	85
24	102

   Consider an equation in which y represents the total cost of dog walking, in dollars, and is expressed as a function of the number of days.
   a. Identify which function has the greater rate of change and explain your reasoning.
   b. Which plan should you choose? Explain your reasoning.

2. Shawna wants to begin offering pet boarding. She is considering charging $32 for 24-hour pet boarding. If she does not board the pet for 24 hours, she will only charge for the number of hours she kept the pet. Her competitor, the Pampered Pet Spa, presents their fee for pet boarding as a table of values, which increases at a constant rate.

Hours	Cost ($)
5	7.50
8	12
11	16.50

   Consider an equation in which y represents the total cost of boarding a pet, in dollars, and is expressed as a function of the number of hours.
   a. Identify which function has the greater rate of change and explain your reasoning.
   b. Suppose you plan on boarding your dog for seven days. Which plan should you choose? Explain your reasoning.

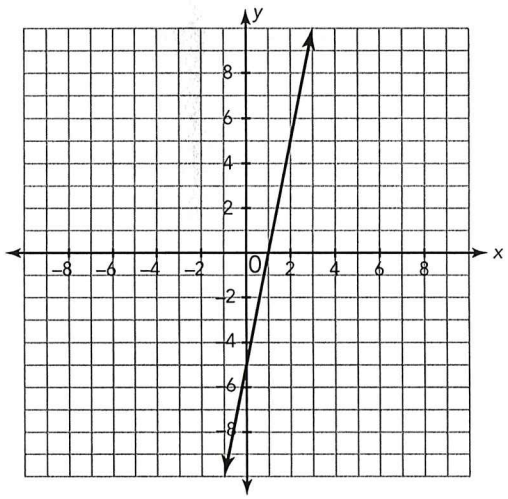
3. Shawna is rethinking her pet boarding business and is considering daily pet boarding that includes play time and regular walks. She is interested in how other businesses charge for similar services. Consider the four companies she researched.

For each company, consider the representation where the dependent value is the total cost, in dollars, to board a pet, and the independent value is the number of hours for the pet's stay.

Beautiful Fur Babies displays the equation $y = 5 + 3x$.

Darling Divas charges $2.75 per hour to board a pet.

Absolutely Perfect Pets:



Cozy Critters:

Hours	Cost ($)
2	7
4	14
6	21
8	28

a. Order the businesses by rate of change. Justify your order.

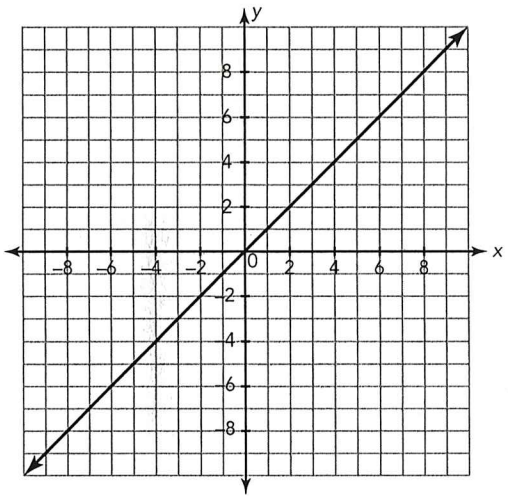
b. If each business requires a two hour minimum stay and a pet owner wants to board a cat for two hours, which business should the pet owner choose?

c. Shawna wants to compete with these local pet boarding businesses. Design a fee schedule for Shawna's pet boarding business.

Stretch

Compare the rates of change for the given functions.

Function A:



Function B: $y = x^2$

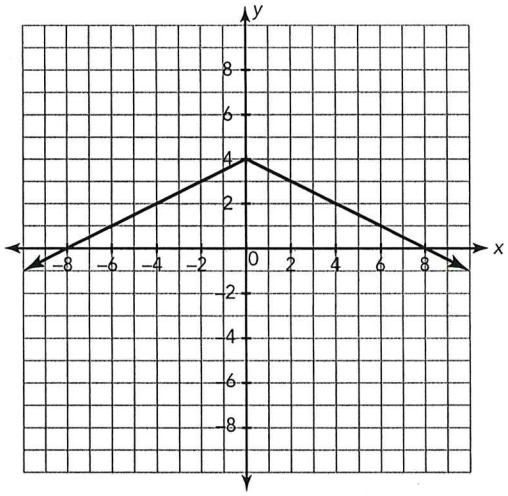
Function C:

x	y
−3	2
−1	0
0	1
1	2
2	3

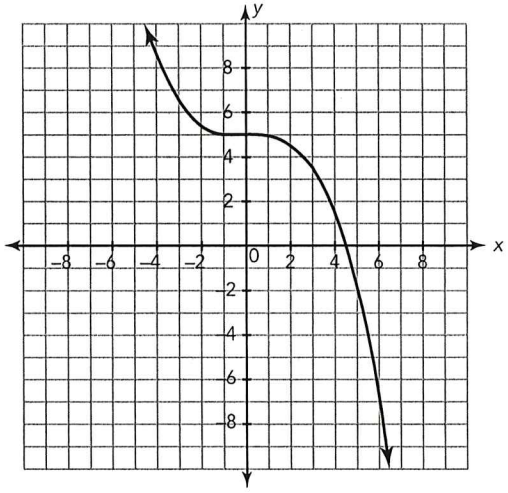
Review

Describe each interval of increase, interval of decrease, and constant interval for the graphs shown.

1.



2.



Write an equation for a line with the given characteristics.

3. Passes through the points $(-3, 17)$ and $(5, -8)$
4. Slope of the line is $\frac{8}{5}$ and passes through the point $(0, \frac{1}{4})$

Determine the slope and y-intercept of the line represented by each equation.

5. $12x + 4y = 24$
6. $-x + 3y = 18$